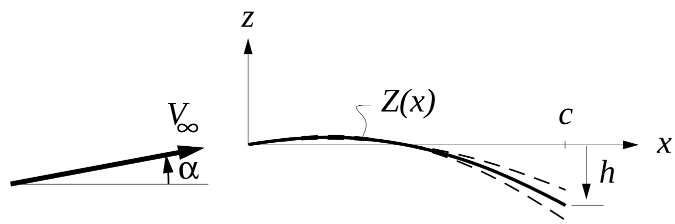
1. (30 %) A thin airfoil has an adjustable camberline defined by

$$Z(x) \;=\; 2h\,\frac{x}{c}\left(\frac{1}{2}-\frac{x}{c}\right)$$



where h is the downward camberline deflection at the trailing edge. The freestream V_∞ is at some arbitrary α as shown.

a) Determine the coefficients A_0, A_1, A_2, … for this camberline and α.

b) Determine the airfoil's c_ℓ and $c_{m_{c/4}}$, as functions of h/c and α.

c) Determine the zero-lift angle $\alpha_{L=0}$, as a function of h/c.

2. (20 %) At a freestream angle of attack of $\alpha = 0.1\,\mathrm{rad}$, the flow around a particular thin airfoil airfoil is represented by the following vortex sheet strength:

$$\gamma(\theta) \;=\; V_\infty \, (1.0\;\sin\theta \;+\; 0.4\,\sin 2\theta)$$

where $x/c = (1 - \cos\theta)/2$ as usual.

a) Determine the airfoil's c_ℓ and $c_{m_{c/4}}$.

b) Determine the camberline slope dZ/dx that the airfoil must have at the midchord location $x = c/2$.

3. (30 %) A wing to be designed is to have the following circulation distribution in level flight:

$$\Gamma(y) \;=\; V_\infty b\,(0.05\,\sin\theta \;-\; 0.005\,\sin 3\theta)$$

where $2y/b = \cos\theta$ as usual. We will assume $\rho{=}1$, $V_\infty{=}1$, $b{=}2$.
Note also: $\sin 3\theta \;=\; \sin\theta\;(4\cos^2\theta - 1)$

a) Determine the lift L and induced drag D_i of this wing.

b) Determine the downwash velocity $w(y)$, and sketch it roughly.

c) The wing is to have a spanwise-constant $c_\ell = 1$. Determine the necessary planform $c(y)$.

d) The wing and fuselage have a common reference axis which is to be horizontal in level flight. Determine the necessary aerodynamic twist distribution $\alpha_{\rm aero}(y)$ relative to this reference axis.

4. (20 %) An elliptically-loaded wing with aspect ratio $AR = 20$ has an airfoil with the following 2D profile drag polar:

$$c_d(c_\ell) = 0.015 + 0.01 c_\ell^2$$

We will assume that $c_\ell = C_L$.

a) Write an expression for the overall drag coefficient C_D of the wing.

b) Determine the minimum power coefficient $C_D / C_L^{3/2}$ that this wing can produce.